Generalised Momentum and Cyclic coordinates:–

let a single Particle, moving with velocity $\dot{x}$ along x-axis.
The ~~Kinectic~~ Kinetic energy of the Particle is

$$T = \tfrac{1}{2} m\dot{x}^2 \quad \text{——— (1)}$$

The derivative of T with respect to $\dot{x}$, i.e

$$\frac{\partial T}{\partial \dot{x}} = m\dot{x} = p \text{ (momentum)}$$

If 'V' is not a function of the velocity $\dot{x}$, i.e $V = V(x)$

and $\frac{\partial V}{\partial \dot{x}} = 0$, then the momentum P can be written as

$$P = \frac{\partial}{\partial \dot{x}}(T - V) \quad \text{or,} \quad P = \frac{\partial L}{\partial \dot{x}} \quad \text{——— (2)}$$

similarly for a system described by a set of generalised coordinates 'q_K' and generalised velocities $\dot{q}_K$.

The generalised momentum corresponding to the generalised coordinates q_K as

$$P_K = \frac{\partial L}{\partial \dot{q}_K} \quad \text{——— (3)}$$

This is also called Conjugate momentum (conjugate to the ~~Conjugate~~ Coordinate q_K) or Canonical momentum.

For a Conservative system,
the Langrange's equations are

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_K}\right] - \frac{\partial L}{\partial q_K} = 0 \quad \text{——— (4)}$$

Substituting for $\frac{\partial L}{\partial \dot{q}_K} = P_K$ (we get)

$$\frac{dP_K}{dt} - \frac{\partial L}{\partial q_K} = 0 \quad \text{or,} \quad \dot{P}_K = \frac{\partial L}{\partial q_K} \quad \text{——— (5)}$$

Now, Lagrangian L of a system, a certain coordinate q_k does not appear explicitly, then

$$\frac{\partial L}{\partial q_k} = 0 \quad \text{—— (6)}$$

This means from eqn (5), that

$$\dot{P}_k = \frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_k}\right] = 0 \quad \text{—— (7)}$$

taking integration, we get

$$P_k = \frac{\partial L}{\partial \dot{q}_k} = \text{Constant} \quad \text{—— (8)}$$

Thus, the Lagrangian function does not contain a coordinate q_k explicitly, the generalised momentum P_k is a constant of motion. The coordinate q_k is called <u>cyclic</u> or <u>ignorable</u>. In other words, the generalised momentum associated with an ignorable coordinate is a constant of motion for the system.